# Teachers' Beliefs About Subtraction Methods in Moroccan Primary Schools: An Exploratory Case Study in Mathematics Education

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## ABSTRACT

This exploratory study investigates teachers' epistemological beliefs and their preferences for subtraction methods in Moroccan primary schools, comparing two stacking techniques: the borrow-and-payback (based on the equal difference principle) and the borrow-and-regroup (based on the exchange principle). Data from 333 participants, including pre-service teachers, trainees, and active teachers, reveal a strong preference for traditional techniques, often justified by the sentiment, "We teach the way we have learned". However, a significant gap in pedagogical diversity is evident, as most participants were unfamiliar with alternative approaches. Teachers' reliance on personal experience and initial training reflects a constructivist approach to teaching. The findings highlight the importance of incorporating diverse subtraction techniques and raising teachers' awareness of them in training programs, fostering collaboration, and bridging the gap between research and classroom practices. Future research should investigate the impact of various methods on student outcomes and examine the cultural and institutional factors influencing teachers' choices.

**Keywords:** Epistemological beliefs – Student Teachers – Subtraction – Primary Education – Mathematics Education.

# 1. INTRODUCTION

Teachers' epistemological beliefs, i.e., their understanding of the nature of knowledge and learning, play a pivotal role in shaping pedagogical practices, particularly in mathematics education, where conceptual understanding and methodological choices are critical (Brownlee, Schraw, and Berthelsen 2011). One area where these beliefs are particularly influential is in the teaching of subtraction, a foundational skill in primary mathematics. The stacking (borrowing) method, where pupils are instructed to 'borrow' from a higher place value to perform subtraction, has long been a dominant approach, especially in contexts emphasizing procedural fluency (Brownell 1947). In Morocco, this method is often perceived as the most effective and contextually appropriate strategy for teaching subtraction, though this perception has not been empirically validated. This study aims to investigate whether teachers indeed hold this view and how it influences their pedagogical decisions.

Emerging evidence, however, suggests that the stacking method may generate difficulties for pupils, depending on the underlying principle guiding the strategy, such as the exchange principle or the equal difference principle (Artemenko et al. 2018). This study explores teachers' epistemological beliefs and their perceptions of two stacking techniques, which may appear similar at first sight but are based on two distinct underlying principles:

(1) Borrow-and-Regroup (BR), an exchange-principle-based stacking technique: Where one unit from the next higher place value of the minuend is exchanged for 10 units in the current lower place value (see Figure 1-a).

(2) Borrow-and-Payback (BP), an equal-difference-principle-based stacking technique: Where 10 units are added to the minuend in the current place value, and 1 unit is added to the subtrahend in the next higher place value (see Figure 1-b).

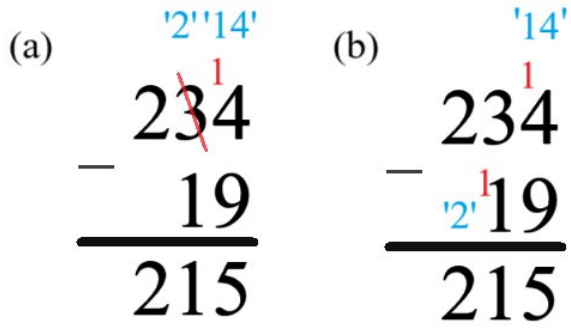


**Figure 1.** Subtraction 234 – 19 illustrated with Borrow-and-Regroup and Borrow-and-Payback techniques.

This study seeks to explore how teachers' epistemological beliefs shape their approaches to teaching subtraction in Moroccan primary schools, addressing key questions about the nature of subtraction knowledge, its acquisition, and its implications for pedagogical practice. Specifically, it examines whether the BR or BP technique is perceived as dominant, and how these perceptions influence teachers' instructional choices and their views on pupils' difficulties in learning subtraction. Drawing exclusively on questionnaire data, this research contributes to understanding teachers' representations and beliefs about subtraction, offering a preliminary analysis that could inform future pedagogical strategies and teacher training programs in Morocco.

## 2. THEORETICAL FRAMEWORK

The research on epistemological beliefs, as described by Hofer (2000), focuses on individuals' beliefs and theories regarding knowledge and the methods by which it is acquired. According to Hofer and Pintrich (1997), personal epistemology consists of two key dimensions: the nature of knowledge and the nature or process of acquiring knowledge. These dimensions are further divided into four sub-dimensions, as structured by Hofer and Pintrich (Figure 2).

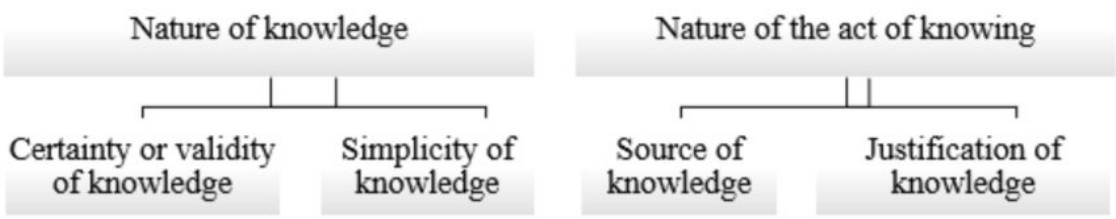


*Figure 2. Hofer and Pintrich Model of Personal Epistemology (1997).*

The framework of this study employs Hofer and Pintrich (1997) four-dimensional structure, recontextualized for mathematics education:

(1) Certainty of Knowledge: Mathematics teaching beliefs exist on a continuum from absolute perspectives (mathematical procedures as fixed, universal truths) to relativist orientations (mathematical methods as contextually adaptable). This dimension manifests in beliefs about whether subtraction algorithms represent immutable procedures or adaptive approaches.

(2) Simplicity of Knowledge: Teachers' conceptions about mathematical understanding range from viewing knowledge as discrete, isolated procedures to integrated conceptual networks. This affects whether subtraction with regrouping borrowing is presented as algorithmic steps or as conceptual understanding of place value and numeration systems.

(3) Source of Knowledge: This dimension encompasses beliefs about whether mathematical pedagogical knowledge originates from external authorities (curriculum, textbooks, tradition) or through personal construction via experience and reflection. It addresses how teachers determine their instructional approach to subtraction.

(4) Justification of Knowledge: This examines the evidentiary standards teachers employ to evaluate instructional effectiveness, including empirical evidence of student outcomes, alignment with mathematical principles, and cultural-pedagogical compatibility.

Drawing on Hofer and Pintrich (1997) four-dimensional model, this study examines how teachers' beliefs about the nature and acquisition of knowledge shape their pedagogical approaches to teaching subtraction in Moroccan primary schools. The research addresses the following questions

(1) What epistemological beliefs do future primary teachers hold regarding the nature of subtraction knowledge (certainty and simplicity)?

(2) How do they perceive the acquisition and justification of subtraction knowledge in the teaching process (source and justification)?

(3) How do these beliefs shape their intended instructional strategies for teaching subtraction in Moroccan public primary schools?

# 3. RESEARCH METHODOLOGY

This section outlines the sample, data collection, and analysis procedures used in this exploratory case study.

## 3.1. Sample

The study included 333 participants from Moroccan primary education and teacher-training contexts. Active teachers (AT, n = 77; 23.1% of the total sample) comprised early-career teachers (<5 years, 34% of AT) and senior teachers (>10 years, 35% of AT). The majority were student teachers (ST, n = 249; 74.8%), enrolled in higher education institutions such as the Higher School of Education and Training (ESEF) and the Higher Normal School (ENS), along with trainees from regional Centres for Education and Training Professions (CRMEF). A small number of participants (n = 7; 2.1%) fell into the "other" category.

## 3.2. Data Collection

Data were collected via a structured online questionnaire distributed through Google Forms, available in Arabic, French, and English (the three main languages used for teaching mathematics at the Moroccan primary level) to ensure comprehension and inclusion. The questionnaire comprised eight main questions, both closed- and open-ended, addressing:

- Professional experience: Years of teaching.
- Preferred method: Choice of subtraction method (stacking or alternatives), with explicit preference between the BR and BP techniques, illustrated and explained to avoid misunderstandings.
- Certainty dimension: Beliefs about whether subtraction procedures are fixed or adaptable.
- Simplicity dimension: Views on whether subtraction is best taught stepwise or as an integrated concept.
- Source dimension: Pedagogical subtraction sources (e.g., curricula or personal experience).
- Justification dimension: Perceived effectiveness of each subtraction method and technique, including student outcomes and alignment with mathematical principles.

The questionnaire was distributed online over three months. Participants responded anonymously to encourage honest answers without fear of judgment. It was pretested with a small sample of prospective teachers and colleagues to ensure clarity and validity. Responses included both quantitative and qualitative data.

### 3.3. Data Analysis

Collected data were analysed using descriptive statistics in SPSS to identify general trends. Qualitative responses were analysed thematically to uncover underlying epistemological beliefs and justifications.

## 4. RESULTS

The data reveals a significant imbalance in the distribution of mathematics teaching experience among respondents. As noted in the Method section, the sample was predominantly composed of student teachers (74.77%), with active teachers representing a smaller proportion (23.1%). This imbalance should be considered when interpreting the findings, as the perspectives of experienced practitioners are underrepresented in the data.

### 4.1. Certainty of the method

The findings indicate that BP is highly dominant among participants (Figure 3-a). Most participants (80.4%, n = 269) reported teaching stacking subtraction primarily using the BP technique, while only 17.8% (n = 59) reported using BR. Each technique user believed their approach was more efficient than the other (Figure 3-b).

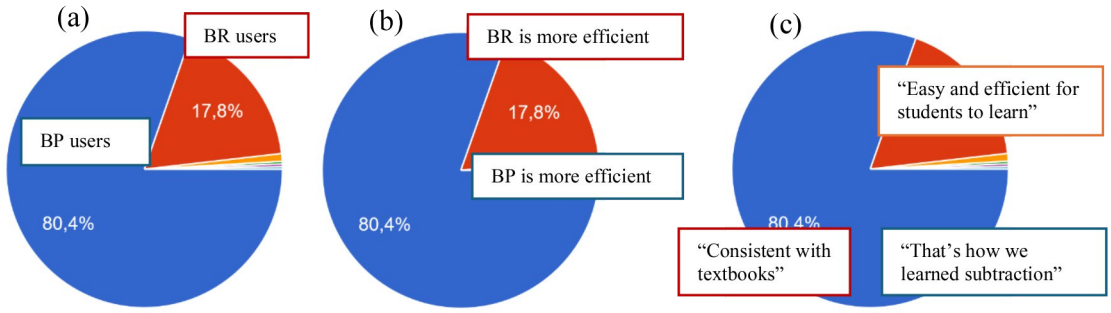


**Figure 3.** Percentage of participants using Borrow-and-Payback versus Borrow-and-Regroup, including justification and perceived pedagogical efficiency.

Both BP and BR users described the reasons for their exclusive use of their selected technique (Figure 3-c). Most participants (58.6%, n = 195) believed their chosen technique was the most efficient and easiest for pupils to learn. A smaller proportion (30.3%, n = 101) cited tradition as their primary reason, stating, "That's how we learned, and it works just fine," while 11.1% (n = 37) believed their technique was the only one aligned with the curriculum, although both are.

Teachers predominantly adopt a relativist perspective: 70.9% (n = 236) asserted that subtraction techniques depend on context, and 87.1% (n = 290) believed stacking techniques can evolve. Ease of pupil understanding (58.6%, n = 195) was the primary reason for choosing the technique, indicating a preference for adaptive, student-centred approaches over rigid, universal procedures. This flexibility aligns with context-driven pedagogy but may lead to variability in instructional consistency.

### 4.2. Simplicity of the method

The majority of participants (64.3%, n = 214) perceived the stacking method as not being a complex procedure, while a smaller proportion (33.3%, n = 111) considered it slightly

complex. Notably, an overwhelming majority (87.1%, n = 290) reported using didactical tools, such as tokens and bonds, to make the stacking method more accessible and less complex for pupils. This reliance on hands-on tools may explain why many teachers viewed the method as simple, despite acknowledging its potential complexity. This suggests an emphasis on conceptual understanding (e.g., place value) over isolated algorithms. The integration of didactical tools likely mitigates perceived learner difficulties, fostering deeper comprehension rather than rote memorization.

## 4.3. Source of pedagogical knowledge

Teachers in the sample relied heavily on personal experience (44.4%, n = 148) and initial training (34.8%, n = 116) as primary sources of pedagogical knowledge. In contrast, external authorities, such as curriculum guidelines and official directions, had minimal influence (13.8%, n = 46), as did experience exchange with colleagues (6.9%, n = 23). This pattern reflects a constructivist approach, where pedagogical knowledge is primarily built through practice and formal education. Reliance on personal experience and initial training may indicate teachers' confidence in their own practices, while the limited impact of external authorities and colleagues suggests gaps in institutional support or collaborative professional development. Although fostering individualized teaching, it may also result in uneven adoption of standardized strategies, despite the observed preference for BP.

Despite the variation in stacking techniques used in practice (BP and BR), all participants (100%, n = 333) reported learning subtraction in primary school using BP. This indicates a strong historical influence of the BP method in Moroccan primary education, shaping teachers' preferences even as they encounter alternative approaches during training. These findings underscore the need to balance individual expertise with institutional support and collaborative professional development to ensure consistency and effectiveness in teaching.

## 4.4. Justification of the method

Teachers primarily judge the effectiveness of subtraction techniques through pupil feedback (48%, n = 160) and assessment results (35.9%, n = 120), highlighting a reliance on empirical validation. In contrast, only a small proportion consider professional exchanges (4.5%, n = 15) or direct in-class observations (11.1%, n = 37). The strong preference for the BP technique, with 76.9% (n = 256) of participants viewing it as reliable and efficient (Figures 3-b and 3-c), reflects its perceived clarity, alignment with teachers' historical experiences, and adaptive, experience-driven practices. The participant quotes (see table 1) further illustrate this alignment.

**Table 1.** Translated participant quotes

| Participant | Original language | Translated from original language to English |
|---|---|---|
| Participant A | French | I learned subtraction using traditional methods like textbooks, where exercises were presented progressively. The teacher explained the theory, gave examples on the board, and provided practical exercises. Additionally, there were supplementary teaching tools like tokens or illustrations that helped me understand the concept concretely. |

| | | |
|---|---|---|
| Participant B | Arabic | I learned subtraction during my studies in a gradual way. Initially, I started by understanding the basic concept of subtraction using finger counting or tokens. Then, I moved on to vertical subtraction with small numbers, where we learned to borrow (BP) when necessary. As I progressed, I practiced subtraction with larger numbers, including decimals and word problems. In advanced stages, I used subtraction to solve equations and more complex problems. |
| Participant C | Arabic | I learned subtraction during my studies by first placing the problem vertically (stacking). If we needed to borrow, we took a ten from the tens place in the second row and then returned the borrowed amount afterward. |
| Participant D | French | We perform subtraction 'normally' if the digit in the first number is greater than the digit in the second number. Otherwise, we add 1 (which represents 10). |
| Participant E | Arabic | I taught myself subtraction by working with tangible objects. |
| Participant F | Arabic | When it comes to vertical subtraction, like 198 – 19, I borrow (BP). |

## 5. DISCUSSION

This study explored teachers' epistemological beliefs regarding pedagogical knowledge for teaching subtraction at the Moroccan primary level, focusing on stacking methods and specific techniques. Findings revealed a strong preference for the BP technique, with 76.9% of participants considering it reliable and highly efficient. This dominance reflects both historical exposure and teachers' reliance on empirically observable pupil outcomes, consistent with a constructivist orientation in which personal experience and initial training shape pedagogical decisions.

The persistence of traditional methods aligns with prior research on the influence of epistemological beliefs. Mardiha and Alibakhshi (2020) reported that teachers' epistemological beliefs strongly shape pedagogical choices, while Tamimy (2015) emphasize that consistencies and inconsistencies can exist between beliefs and practices, highlighting the need for further investigation. Lammassaari et al. (2022) found that teachers' epistemic beliefs significantly affect pedagogical work across cultural contexts. Domain-specific studies indicate that epistemological beliefs in mathematics differ from other domains (Urhahne and Kremer 2023) and develop in relation to more general beliefs (Korom et al. 2023). Guven (2012) further underscore the importance of considering variations in epistemological beliefs across fields when designing educational reforms.

Despite all participants having learned subtraction through BP during their own schooling, a significant proportion (90.6%) remained unfamiliar with alternative methods, such as BR, even after completing teacher training programs (ENS, ESEF, CRMEF) or gaining teaching experience. This highlights a narrow pedagogical repertoire and a potential gap in instructional diversity, which may limit the strategies offered to pupils and influence their

mathematical reasoning. Local research by Ennassiri, Abouhanifa, and Elkhouzai (2025) underscores the critical role of early algebraic thinking in primary education, showing that activities promoting generalization and symbolic reasoning are essential for developing algebraic competencies. Consequently, employing a variety of subtraction techniques, grounded in different mathematical principles and reasoning modes, can support learners in developing algebraic thinking by encouraging abstraction and generalization (Squalli et al. 2020). The persistence of BP also reflects some resistance to pedagogical innovation, consistent with findings by Klopp, Krause-Wichmann, and Stark (2023), who show that targeted interventions can mitigate the influence of entrenched epistemological beliefs, and Shultz et al. (2022), who observed similar contradictions among STEM teachers balancing cultural relevance with "culture-free" disciplinary views.

Teachers primarily judged the effectiveness of subtraction techniques through pupil feedback (48%) and assessment results (35.9%), reflecting a reliance on empirical validation. Only a small proportion relied on professional exchanges (4.5%) or direct classroom observation (11.1%), indicating a gap in collaborative and reflective practices. This pattern reflects a constructivist orientation, where pedagogical knowledge is built through practice and reflection rather than external directives. The persistence and dominance of the BP method, aligned with teachers' historical experiences and pupil outcomes, further underscores the limited exposure to alternative methods. These findings resonate with prior research showing that teachers often rely on methods experienced during their own schooling (Brownlee, Schraw, and Berthelsen 2011), while also highlighting the importance of integrating multiple strategies to address diverse learner needs (Artemenko et al. 2018).

These findings have important implications for teacher training and professional development. Programs should encourage exploration of alternative techniques, such as BR, and promote collaborative practices, including peer observation and professional exchanges. Curriculum designers should provide clear guidance for early-career teachers to ensure consistency and effectiveness in subtraction instruction. Inspectors and teacher trainers can play a proactive role in supporting innovation and bridging the gap between research and classroom practice. Implementing these initiatives may require addressing systemic challenges, such as limited resources, time constraints, and resistance to change among educators.

This study has several limitations. The sample was predominantly composed of pre-service teachers (70.8%), limiting the generalizability of findings to experienced practitioners. Reliance on self-reported data introduces potential social desirability bias. Additionally, the study focused exclusively on teachers' perspectives without examining student outcomes or classroom implementation, which could provide a more complete understanding of the effectiveness of different subtraction methods.

Future research should address these limitations by examining the impact of different subtraction techniques on pupil learning outcomes, particularly in diverse classroom settings. Longitudinal studies tracking changes in teachers' epistemological beliefs and pedagogical practices over time could yield insights into factors shaping professional development. Comparative studies across educational contexts could further illuminate the cultural and institutional factors influencing teachers' technique preferences.

## 6. CONCLUSION

This study highlights Moroccan primary school teachers' strong preference for traditional subtraction methods, notably the stacking borrowing techniques, shaped by personal

experience and historical practices, reflecting a constructivist orientation in which knowledge is built through practice and reflection. Despite this, a significant gap exists in their familiarity with alternative approaches, limiting pedagogical diversity and potentially constraining pupils' mathematical reasoning and early algebraic thinking. The findings call for teacher training programs to integrate diverse subtraction strategies, foster collaboration through peer exchanges and reflective practices, and bridge the gap between research and classroom implementation. Future studies should investigate the impact of various techniques on student learning outcomes and examine cultural and institutional factors influencing teachers' choices, ultimately supporting more effective, evidence-based mathematics education in Morocco.

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