Empowering teachers as drivers of educational change: The role of teachers' knowledge in achieving inclusive and equitable high-quality mathematics education

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Abstract

Grounded in Senge's change theory, this study explores the role of teachers' knowledge in supporting equitable, rigorous mathematics education. Rigor in math includes three aspects: conceptual understanding, procedural fluency, and application (CCSS, 2010). A semi-structured questionnaire was administered to 42 U.S. PK–12 mathematics teachers, with 38 responses. The study explored teachers' knowledge of the distinction between conceptual understanding and application, as well as their familiarity with the Standards for Mathematical Practice (SMPs). Findings show that 12 participants accurately distinguished conceptual understanding from application. However, 26 agreed that solving real-world problems indicates conceptual understanding, revealing a possible misinterpretation or overlap between the two. Only one respondent listed all eight SMPs, while most (n = 35) could not list them or skipped the question. When asked to differentiate SMPs from content standards, six participants did so correctly, four partially, and 28 were unable to distinguish them. These results highlight gaps in teachers' knowledge of mathematical rigor and practice standards. Although the study is based in the U.S., it reflects global challenges. Education leaders worldwide should consider how teacher knowledge, curricular standards, and systemic support align to advance equity and rigor in mathematics education.

Keywords: equity inclusion quality mathematics education SDG4 teachers' knowledge

Introduction

Global leaders at the 2015 World Education Forum emphasized education's transformative power to advance the United Nations (UN) Sustainable Development Goals (SDGs). Among them, Goal 4 (SDG4) emphasizes the need for all students to have access to equitable, quality education that enables literacy, numeracy, and the skills necessary for upward mobility, helping address workforce inequalities (UNESCO, 2017). This global agenda is causing changes in curricula and pedagogical practices in schools to help prepare youth with the skills needed to work and live in the 21st century (Snyder, 2023). In math education, changes in teaching and learning need to shift from rote memorization to conceptual understanding (NCTM, 2023) to better prepare youth with needed skills in numeracy. The National Science Foundation projects that 80% of future jobs will demand math and science expertise (NSF, 2020), in which numeracy, the mathematical ability to apply mathematical reasoning in real-world contexts, is central (PISA, 2022).

According to Pink (2005), we live in the Conceptual Age, which demands that individuals think critically, solve problems, and transfer ideas effectively. With the exponential growth of information and the digital revolution, success increasingly depends on efficiently processing new information and engaging in higher abstraction levels (Wathall, 2016). In mathematics education, this shift warrants a focus on learning for understanding rather than mere performance (Wathall, 2016). Larson (2017) emphasized that to be mathematically literate in the 21st century, students must understand *how* to perform mathematical procedures, *why* they work, and *when* they should be applied. Achieving this goal demands a systemic shift in teaching practices for all educators. The *Principles to Action* document advocates moving from isolated excellence to systemic excellence, emphasizing comprehensive, targeted professional development in mathematics instruction (NCTM, 2014) for teachers to become part of the driving force to transform math education. Effective teachers play a pivotal role in preparing students to connect mathematical concepts with real-world contexts, aligning their teaching practices with the global mission of SDG4 to promote educational equity and sustainability (Association of Mathematics Teacher Educators, 2017).

Equal access to high-quality, rigorous math education becomes a critical question for educators, who need to ensure that the quality of education is in line with the contemporary needs of society. Quality education involves not only accessibility but also instructional rigor. Rigor in mathematics education is composed of three aspects: conceptual understanding, procedural fluency, and application of knowledge to real-world scenarios (Common Core State Standards [CCSS], 2010). Studies show that teachers' knowledge and instructional approaches are not always aligned with the evolving demands of mathematical literacy, particularly vis-à-vis the integration of conceptual understanding, procedural fluency, and real-world applications, which are critical for preparing students for the 21st century (NCTM, 2023). For teachers to effectively guide students toward attaining this conceptual understanding, they must have a clear comprehension of its meaning; "If teachers do not understand the concept-based model and require shifts in pedagogy, they will fall back on traditional teaching methods and fail to affect the transfer of knowledge and deep understanding" (Erickson, 2012, p.10). If teachers do not possess the necessary knowledge and understanding of the shift in mathematics education, they become gatekeepers that hinder progress toward the achievement of SDG4 (International Labour Organization, 2023), rendering them ineffective to serve as drivers of educational change (UNESCO, 2023).

The purpose of this article is to present findings from a study exploring teachers' perceived knowledge of rigor in mathematics education—specifically their grasp of the distinction between conceptual understanding and the application of mathematics. It also explores teachers' familiarity with the Standards for Mathematical Practice (SMPs), which outline the behaviors and habits of mind students must develop to achieve conceptual understanding, procedural fluency, and mathematical application. A solid understanding of both rigor and practice standards is essential for driving meaningful change in mathematics education. The study is delimited to the U.S. context, but the challenges it reveals are globally relevant. Education leaders worldwide should consider how teacher knowledge, curricular standards, and systemic support align to advance equity and rigor in mathematics instruction. Senge's theory of profound change served as an analytical framework for identifying potential barriers stemming from gaps in teachers' knowledge. By making these gaps visible, the findings aim to inform targeted areas for professional development that support systemic and sustainable progress in mathematics education.

Theoretical Background

Change in education

The UN Secretary General's High-Level Panel on the Teaching Profession highlights the need to empower teachers as drivers of educational change (UNESCO, 2023). For the empowerment to be effective, teachers need to possess both the knowledge and the mindset that underpin the contemporary approach to math education. For many teachers, this will require a mind-shift (Boaler, 2018-19) from how they were raised and trained to teach math based on rote memorization to help students develop a mathematical mindset. Further, Garvin (2000) underscores that knowledge is not merely a theoretical construct but a capacity for effective action. In mathematics education, this means teachers need a deep understanding of mathematical concepts to promote meaningful student comprehension and application. Schools, therefore, must create conditions for teachers to master both content and pedagogy, ensuring that students receive a quality education that meets societal needs.

The educational change process is complex (Senge et al., 1994; Snyder et al., 2008) and often lacks focus on the importance of helping teachers develop the necessary knowledge to serve as change agents. For example, studies in transforming education through digital technology repeatedly demonstrate the lack of sufficient knowledge among teachers needed to transform their teaching and learning practices (Snyder et al., 2023). Instead, teachers rely on technology specialists, which results in short-lived projects (Fischer et al., 2020). In other studies, Ab Kadir (2016) described the complex nature of teacher knowledge as more than subject specific. Teacher knowledge includes subject knowledge, content knowledge, pedagogical knowledge, and, ultimately, pedagogical content knowledge. Often lacking is a focus on professional development needed to provide teachers with both the knowledge and understanding of implications for teaching and learning (Brandt, 2022).

Transforming schools requires internal processes that support continuous growth through knowledge sharing, reflection, and dialogue that generate shared mental models and organizational growth (Senge et al., 1994; Snyder, 2023). Snyder et al. (2008) report on schools and school districts across the globe that have transformed their cultures using a systems approach involving shared visioning, dialogue and reflection, teacher empowerment, ongoing professional development, and informed decision-making. In Sweden, Björkman (2008) found that when teachers and principals had a shared vision, knowledge, and views of the school, the internal capacities for sustaining transformation were improved. These studies and others like them reinforce the importance of a systems approach in schools and the need for teachers and principals to develop shared knowledge and a culture of learning if sustainable change is to occur.

Profound Change

Senge (1999) raises important questions about the change process. Sustaining any profound change process requires a fundamental shift in thinking. Stakeholders need to understand both the forces that support growth and those that impede progress and develop strategies for addressing these obstacles. He describes this dynamic interplay as the “dance of change,” where "growth processes" (forces driving positive change) interact with "limiting processes" (barriers hindering change) to shape the trajectory of organizational transformation (Senge, 1999). These changes are driven by meaningful learning at both individual and organizational levels which is supported by Systems Thinking. Grounded in theories of feedback and complexity, systems thinking explains how stability or growth develops over time (Senge, 2012). It also emphasizes viewing organizations as networks of interconnected parts rather than isolated elements, which supports the development of a shared purpose.

Senge (1999) articulated five key disciplines for supporting organizational transformation. These include personal mastery, mental models, shared vision, team learning and systems thinking. Personal mastery reflects the gap between current reality and personal vision; mental models reflect deeply ingrained assumptions within individuals and organizations; a shared vision is a collective desire that acts as a compass for change; team learning grows from cultures of shared dialogue and reflection, and systems thinking integrates the first four disciplines.

In this article Senge's theory of profound knowledge is used as a theoretical lens to explore implications for change based on teacher's knowledge. One of the key dimensions, personal mastery is used to represent the individual contribution of teacher to the change process. While the remaining dimensions are included in the full-scale model to represent a system of change, this article focuses specifically on personal mastery. In education, change is often directed by policy without raising the question if educators possess the necessary knowledge. To address this gap, this paper focuses specifically on the knowledge of teachers to the exclusion of analyzing the broader aspects of the system. This decision was based on the premise espoused by the UN (UNESCO, 2023) that teachers need to be empowered as drivers of change. For that to happen, they need minimum knowledge about what is asked of such change.

Mathematical Literacy (Numeracy): A Mind Shift

The National Council of Teachers of Mathematics (NCTM) advocates three essential purposes for learning mathematics: developing deep mathematical understanding; learning to understand, question, and critique the world, including its injustices, through mathematics; and experiencing the wonder, joy, and beauty of mathematics (NCTM, 2023). By fostering a deep conceptual understanding of mathematics among students, we enhance their career readiness and contribute to the broader goals of economic growth, national security, and global competitiveness. To support this agenda, mathematics education goals have evolved from a two-dimensional model, mile-wide and inch deep, where the focus is on facts and skills, to a three-dimensional model, where the importance of conceptual understanding is recognized. This model's goal is to create deep knowledge, transferable understanding, and higher-order thinking while topics, facts, and skills remain essential components of the teaching/learning process (Erickson, 2012). To better understand the 3D model for mathematics education, it is necessary to distinguish between the three aspects of rigor: conceptual understanding, procedural fluency, and the application of mathematics.

Conceptual Understanding, Procedural Fluency, and the Application of Mathematics

This section provides the reader with a basic understanding of what is meant by teaching for conceptual understanding and the Standards for Mathematics Practice. While national curricula vary, each country adopts their own standards, strands, or frameworks that serve as essential reference points for instructional practice. Regardless of context, it is critical for teachers to develop a clear understanding of the guiding principles specific to their national setting. Examples include Australia's Proficiency Strands (ACARA, 2022), Singapore's Mathematical Processes (Singapore Ministry of Education, 2012), and Canada's Mathematical Processes Framework (Ontario Ministry of Education, 2005). In this study, the U.S. Standards for Mathematical Practice (SMPs) are used as the primary framework. The dimensions presented below formed the basis for the development of the questionnaire employed in this research.

Conceptual Understanding

Conceptual understanding is an integrated and functional grasp of mathematical ideas (National Research Council, 2001). Students who have a conceptual understanding of mathematics know more than isolated facts and methods and see mathematics as more than a set of mnemonics or discrete procedures (Michalek, 2019). This approach to mathematics contrasts with rote memorization, which has been prevalent in schools worldwide for decades. The rote memorization approach reflects schools' culture of indoctrination that focuses on teaching procedures without any connection to their meaning, which limits students' understanding and the practical application of these procedures (Huinker & Bill, 2017; NCTM, 2014). Therefore, mathematics educators must continually work to help students make sense of mathematics beyond memorizing a series of steps (Dykema, 2022).

A key sign of conceptual understanding is when a student can explain why a math statement is true or where a rule comes from, in a way that fits their academic level. There's a big difference between a student who just uses a trick, for example, the FOIL method, to expand (a + b) (x + y) and one who can explain why the trick works. FOIL is a mnemonic that stands for First, Outer, Inner, Last, helping students remember the steps to multiply binomials. However, understanding involves recognizing that each term in the first binomial is distributed to each term in the second binomial. The student who can explain it really understands the math and might do better with new problems like (a + b + c) (x + y). Conceptual understanding and procedural fluency are equally important, and teachers can assess both by using mathematical tasks of sufficient richness (CCSS, 2010).

Procedural Fluency

Procedural fluency encompasses the ability to perform procedures efficiently, adaptably, and accurately. This involves transferring procedures to various problems and contexts, constructing or modifying existing procedures, and discerning which strategy or procedure is best suited to a particular situation (NCTM, 2014; National Research Council, 2001). Teaching procedural fluency empowers students to make reasoning and decision-making central to instruction (NCTM, 2023). By receiving education based on such teaching methods, students no longer wonder, *How did my teacher show me how to do this?* Instead, they ask, *Which of the strategies that I know are a good fit for this problem?* This paradigm shift indicates students' mastery of procedural fluency and mathematical agency, essential achievements in K–12 mathematics (NCTM, 2023).

Application of Mathematics

The application of mathematics serves as the third component for achieving the necessary changes in mathematics education to support SDG4's objective of fostering mathematical literacy. Mathematical literacy is the ability to apply mathematical reasoning in real-world contexts and is a critical skill for informed citizenship in the 21st century (OECD, 2018). Real-world application is a key component of mathematics education; however, teachers must carefully implement tasks that promote reasoning and problem-solving. One ineffective classroom practice is asking students to mechanically learn simple procedures and solve similar word problems to practice these procedures (Barshay, 2016). Teachers may inaccurately assume that they promote conceptual understanding when allowing students to solve *real-world* word problems. In a study OECD concluded that students who are taught to solve small everyday problems using *tips and tricks* are not good at transferring that knowledge to other contexts (Barshay, 2016). However, when students have a conceptual understanding of mathematics, they can extrapolate (Barshay, 2016). Data from the OECD's PISA revealed that exposure to formal mathematics, which emphasizes deep understanding and abstract reasoning, is a stronger predictor of performance than exposure to applied mathematics or word problems (Schmidt, 2014).

Standards for Mathematical Practice (SMPs)

The authors of the Common Core State Standards (CCSS, 2010) compiled the SMPs using the NCTM's five-process standards and the National Research Council's (2001) standards of mathematical proficiency, resulting in a list of eight SMPs. They focus on what students should do to become proficient in mathematics and outline the behaviors and habits of the mind for students to engage in mathematical content to foster conceptual understanding and procedural fluency. The SMPs are:

- SMP1: Make sense of problems and persevere in solving them
- SMP2: Reason abstractly and quantitatively
- SMP3: Construct viable arguments and critique the reasoning of others
- SMP4: Model with mathematics
- SMP5: Use appropriate tools strategically
- SMP6: Attend to precision
- SMP7: Look for and make use of structure
- SMP8: Look for and express regularity in repeated reasoning

The eight SMPs underscore that mathematics extends beyond calculations (Smith, 2004). Conceptual understanding and application are strands of mathematical proficiency, as reflected in the SMPs. Students need to apply their knowledge to new and accessible situations that extend their understanding (Michalek, 2019). For example, the ability to think abstractly and quantitatively (SMP2) empowers students to progress further in their mathematical education (Smith, 2004), supporting SDG4's objective of promoting lifelong learning opportunities for all.

Teachers must help students develop these mathematical practices to become effective mathematicians (Rutherford, 2015). When teachers give students opportunities to develop the expertise described by the SMPs, they allow them to achieve conceptual understanding (Smith, 2004). This emphasizes SMPs' potential to ensure equitable access to learning experiences for all students, thus positively influencing their future paths (Smith, 2004). According to the Association of Mathematics Teacher Educators (2017), well-prepared mathematics teachers must understand the concepts they will teach and connect those concepts to mathematical practices.

Methods

A semi-structured questionnaire (Creswell & Clark, 2018) was developed to explore teachers' knowledge and perceptions regarding two aspects of mathematical rigor—conceptual understanding and application—as well as their familiarity with the U.S. Standards for Mathematical Practice (SMPs). The questionnaire included both subjective items (e.g., perceptions of rigor) and objective items (e.g., listing the SMPs), offering insight into both perceived and demonstrated knowledge. While the questionnaire draws from established frameworks (see background), it was not designed or validated as a formal assessment tool. As such, it was not subjected to psychometric testing for validity or reliability. This study was exploratory in nature, and findings should be interpreted with caution, acknowledging that the questionnaire offers a snapshot of teacher understanding rather than a definitive measure of knowledge. Teachers from three schools, public and private, were asked to respond to the survey. Convenience sampling, based on a non-probability sampling technique (Creswell & Clark, 2018), was used to select participants based on their availability and willingness to participate.

Data Collection

A semi-structured questionnaire was developed based on the dimensions of mathematics education rigor: conceptual understanding and application, as well as the U.S. Standards for Mathematical Practice outlined in the schools' official mathematics documents.

The questionnaire was to target PK–12-mathematics teachers and included three questions[1] questionnaire targeted PK–12-mathematics teachers and included three questions:

Question 1: Conceptual Understanding	**Questions 2 and 3: SMPs**
This question explored teachers' perceived understanding of the distinction between conceptual understanding and the application of mathematics based on a three-point scale agree/disagree/other.	These questions identified teachers' recognition of the SMPs. Question 2 asked the teachers to list the SMPs from memory. If they did not know the standards, they were instructed to write *I do not know*. Question 3 was multiple-choice, and the teachers were asked to identify the SMPs that were listed among the following content domains: Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations – Fractions, Measurement and Data, Geometry, Ratios and Proportional Relationships, The Number System, Expressions and Equations, Functions, Statistics and Probability

The questionnaire was administered through the SurveyMonkey platform with *Anonymous Responses to ensure confidentiality*. Participants were informed of the anonymity and voluntary nature of the questionnaire before participation. Results were presented at the aggregate level and did not include identifying respondents' information. 38 out of 42 teachers responded, resulting in a response rate of approximately 90%, a high rate that could be influenced by convenience sampling.

Data Analysis

Quantitative data were analyzed using descriptive statistics to summarize teacher responses across the three main questions. For Question 1, responses were categorized into agree, disagree, or other, and frequencies were calculated to identify overall trends. Open-ended responses from those who selected “Other” were analyzed qualitatively using a basic thematic coding approach. Responses were read multiple times to identify recurring patterns, and codes were assigned inductively. Three key themes emerged: distinction between application vs. conceptual understanding; transferability and flexibility; limitations of real-world problems. This qualitative analysis provided deeper insight into teacher thinking and offered nuance beyond the agree/disagree categories.

For Question 2, the number of correctly listed SMPs was counted for each respondent as a measure of familiarity with the standards. In Question 3, responses were scored based on the number of correctly identified SMPs and the accuracy in distinguishing them from unrelated content standards. Finally, to explore broader implications for instructional change, results were interpreted through two dimensions of Senge’s (1994) theory of profound change.

Results

Findings related to teachers perceived understanding of the distinction between conceptual understanding and application are presented first, followed by results concerning their ability to list and recognize the Standards for Mathematical Practice (SMPs).

Mathematical Rigor: Conceptual Understanding and Application

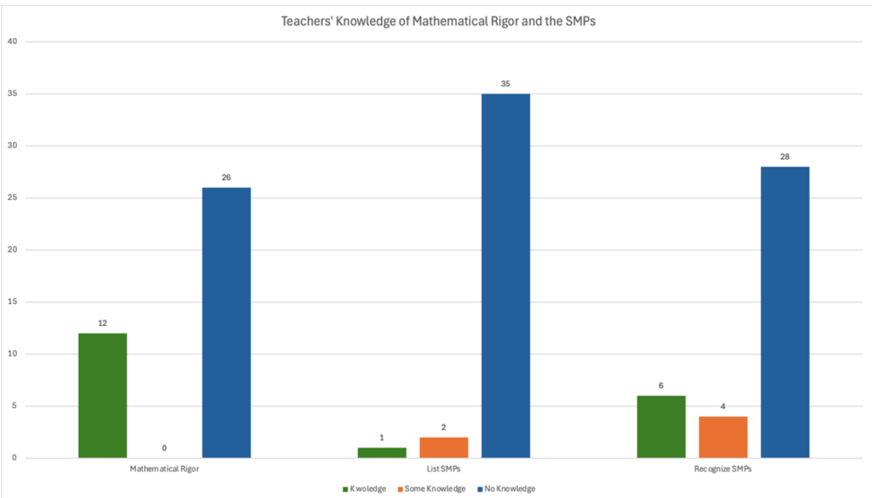
In the questionnaire, mathematics teachers were asked to agree or disagree with the statement, "When students can solve real-world (word) problems, it means that these students have a conceptual understanding of the mathematics needed to solve that problem." Among 38 participants, 26 agreed with the statement, seven disagreed, and five provided an alternative response labeled as *other*, accompanied by written explanations. Concerning the *other* responses, these teachers grasped the distinction between conceptual understanding and application. However, the answers were more complex than mere agreement or disagreement. For example, one person wrote, "I think it is helpful to determine if conceptual understanding has been reached, but it is not always the case. Sometimes students can understand generic/broad concepts but have challenges applying them to real-world scenarios." Another respondent wrote, "I feel as though there are students who can solve real-world problems because they understand the underpinnings of the problem, even if they do not fully understand the concepts." Upon further analysis, their responses (all five) indicated that they recognized the following:

- Application vs. Conceptual Understanding: Some responses suggested that solving real-world problems might indicate procedural knowledge or pattern recognition, especially among students who may not fully grasp the underlying concepts.
- Transferability and Flexibility: Some teachers acknowledged that true conceptual understanding involves transferring and applying concepts to new situations, not just solving specific problems.
- Limitations of Real-World (Word) Problems: Some teachers noted that abstract or pure mathematical concepts do not always apply to real-world scenarios, indicating that conceptual understanding relates to understanding within mathematical concepts.

Recognizing the SMPs

In the second part of the questionnaire, teachers were asked to list the SMPs and distinguish them from the content domains based on a pre-written list. The questionnaire findings are presented in Figure 1, which illustrates the distribution of teachers' demonstrated knowledge across three key areas—mathematical rigor (conceptual understanding versus the application of mathematics), listing SMPs, and recognizing SMPs. The x-axis categorizes the focus area, and the y-axis represents the number of teachers in each knowledge category—knowledge (green), some knowledge (orange), no knowledge (blue).

Figure 1. Teachers' Knowledge of Mathematical Rigor and the SMPs



The data highlight significant knowledge gaps among the respondents:

- **Mathematical Rigor (Conceptual Understanding vs. Application):** Most respondents (n = 26) demonstrated challenges distinguishing between conceptual understanding and application of mathematics, as indicated by the predominance of the blue bar in this category. Only 12 respondents were categorized as knowledgeable (green bar). Of these, seven explicitly disagreed with the statement, "When students can solve real-world (word) problems, it means that these students have a conceptual understanding of the mathematics needed to solve that problem," thereby recognizing that procedural success in context does not necessarily reflect conceptual understanding. An additional five selected "Other" and provided written explanations that reflected an understanding of the distinction.
- **Listing the Standards for Mathematical Practice (SMPs):** Only one respondent successfully listed all eight SMPs (green bar), while two respondents were able to list some (orange bar). In contrast, 27 respondents were unable to list any SMPs. An additional eight respondents skipped the question. All 35 of these individuals were classified as not knowledgeable and are represented by the blue bar.
- **Recognizing the Standards for Mathematical Practice (SMPs):** Only six respondents correctly identified all eight SMPs (green bar), and four recognized some (orange bar). Twenty-two respondents were classified as not knowledgeable because they either did not identify any SMPs or selected items that included both SMPs and unrelated content domains (e.g., Geometry, Number and Operations in Base Ten), suggesting a lack of differentiation between practice standards and content standards. An additional seven respondents did not answer the question and were similarly categorized as not knowledgeable. In total, 29 respondents are represented by the blue bar in this category, indicating a significant gap in knowledge related to the recognition of the SMPs.

Analysis and Discussion

The purpose of this article was to present findings from a study exploring teachers' perceived knowledge of rigor in mathematics education—specifically their grasp of the distinction between conceptual understanding and the application of mathematics. It also explored teachers' familiarity with the Standards for Mathematical Practice (SMPs), which outline the behaviors and habits of mind students must develop to achieve conceptual understanding, procedural fluency, and mathematical application.

This study revealed that 26 out of 38 teachers equated solving real-world (word) problems with conceptual understanding. This perception reflects a limited view of concept-based mathematics teaching, as students can often solve word problems by memorizing procedures without fully grasping the underlying concepts. Several factors may contribute to this misconception. First, mathematical literacy is often defined as the ability to apply mathematical reasoning in real-world contexts and is considered a critical skill for informed citizenship in the 21st century (OECD, 2018). This framing may position word problems as indicators of higher-order thinking, reinforcing the belief that application automatically reflects deep understanding. However, as Barshay (2016) notes, one ineffective yet common classroom practice involves students mechanically learning procedures and applying them to repetitive word problems, which can mask conceptual gaps. As a result, real-world problem-solving may be interpreted as evidence of conceptual depth when, in reality, it may still reflect surface-level reasoning. Second, many teachers may lack professional development that clearly distinguishes procedural fluency, application, and conceptual understanding.

The findings also highlighted a gap in teachers' knowledge of the Standards for Mathematical Practice (SMPs). Only one teacher could accurately list all eight SMPs, and just six could identify them within mathematics content domains. This finding is particularly concerning given that all participating schools had the SMPs outlined in their official curriculum and instructional documents. The presence of these standards in school documentation suggests that accessibility to the information may not be the core issue—instead, the problem may lie in the disconnect between policy and practice. This gap raises questions about the effectiveness of professional development and internal communication strategies within schools. If teachers are unaware of or do not understand these standard practices, they are unlikely to integrate them into their instructional decision-making.

The identified gaps in teachers' knowledge may hinder their ability to provide equitable access to high-quality mathematics education, as solid understanding of rigor and the practice standards is essential for driving meaningful change in mathematics education. Based on the theory of profound change (Senge et al., 1999), individual and shared knowledge is essential for contributing to growth in any organization. The lack of personal mastery of the distinction between conceptual understanding and application in math, as well as the SMPs, suggests that teachers are not yet equipped with the skills and knowledge to contribute to the changes needed to address SDG 4, specifically numeracy. As such, teachers cannot be expected to serve as drivers of change, which is a needed role articulated by the UN Secretary General's Panel on the Teaching Profession (UNESCO, 2023).

These findings reinforce the need for professional development that supports teachers in developing a clear vision of what mathematics should be and how it should be taught. According to Senge’s theory, personal mastery involves a lifelong commitment to learning and self-improvement—an essential component of achieving profound change (Senge et al., 1999). Establishing this vision requires a strong foundation in key mathematics education frameworks, enabling teachers to move from rote memorization toward a concept-based, three-dimensional approach. Without this foundation, teachers may revert to traditional practices (Eriksson, 2012), undermining efforts for change (Senge et al., 1994) and progress toward SDG 4 (International Labour Organization, 2023). Profound change is a gradual process shaped by the interplay of systems thinking (e.g., concept-based mathematics), knowledge (e.g., the SMPs), and intrinsic motivation—the internal “why” that inspires teachers to evolve their practice (Deming, 2018; Senge et al., 1999). To become agents of change (UNESCO, 2023), educators must reflect on their practices, embrace innovative approaches, and develop deep understanding of foundational frameworks such as concept-based teaching, procedural fluency, and the mathematics content and practice standards—avoiding a fallback to outdated, drill-based models of instruction.

Although personal mastery is essential, individual improvement alone is insufficient to drive systemic change based on a systems perspective (Senge, 1999). Team learning—one of the five disciplines—underscores the importance of collaboration in supporting growth processes that address limiting barriers. It invites teachers to reflect together, share knowledge, and align their practices with effective instructional strategies (Senge et al., 1994; Snyder et al., 2008). This approach aligns with total quality management principles, which emphasize the value of collaboration and shared responsibility in achieving continuous improvement (Deming, 2018; Snyder, 2023). Through collaboration, teachers can create a cohesive approach to mathematics instruction that bridges knowledge gaps and ensures students have equitable access to high-quality education. Addressing these gaps requires cultivating both individual and collective growth processes, ensuring teachers are equipped with the knowledge, skills, and mindset to achieve systemic excellence in mathematics education.

While this study offers valuable insights, several limitations must be acknowledged. First, although the research is grounded in the U.S. context through the Standards for Mathematical Practice (SMPs), the findings may not be directly generalizable to international contexts where different educational standards apply—such as Australia's Proficiency Strands, Singapore's Mathematical Processes, or Canada's Mathematics Framework. Future research should explore how teachers in other settings understand and implement their respective national standards, examining how teacher knowledge aligns with curricular expectations across diverse systems. Second, while the questionnaire was effective for collecting broad data, it may not have fully captured the depth and complexity of teachers' knowledge and perceptions. Some items, particularly those addressing conceptual understanding, could have been misinterpreted, potentially introducing variability in the responses. The use of a simple agree/disagree format—even with an "other" option for elaboration—may have oversimplified complex beliefs. Qualitative methods, such as open-ended questions, interviews, or classroom observations, might better capture the nuances of teacher thinking in future studies. Another limitation is the absence of formal reliability and validity testing for the questionnaire. While the questionnaire was grounded in established constructs (e.g., mathematical rigor and the SMPs), it was not designed or tested as a standardized assessment of knowledge. The study was exploratory in nature and aimed to identify broad trends in teacher understanding rather than produce definitive or generalizable measures. Future research could strengthen these findings by employing validated questionnaires or using multiple methods to triangulate teacher knowledge and beliefs. Finally, the use of convenience sampling limits the generalizability of the results. Participants were selected based on availability and willingness, which may not reflect the broader population of PK–12 mathematics teachers. A more representative sample would enhance the applicability of the findings to wider educational contexts.

Looking ahead, future research could explore how additional elements of Senge's framework—particularly systems thinking—might be applied to better align educational policy with classroom practice. A systems-oriented approach may offer strategies for addressing structural barriers and fostering sustainable change in mathematics instruction. Moreover, while this study focused on teachers' knowledge of conceptual understanding and the SMPs, future research could benefit from a deeper exploration of teachers' belief systems. Hofer and Pintrich's (1997) Model of Personal Epistemology—which includes beliefs about the certainty, simplicity, source, and justification of knowledge—offers a valuable framework for examining how teachers perceive and enact mathematics instruction. Investigating these epistemological beliefs could help explain why some educators continue to rely on procedural teaching, even when curriculum frameworks emphasize conceptual understanding. Such inquiry may also enrich our understanding of how deeply rooted mental models, as described by Senge, support or hinder pedagogical change. Although situated in a U.S. framework, the challenges highlighted in this study are not unique to one national context. A key insight emerges: if educators are unaware of, or do not fully understand, the standards and frameworks they are expected to implement, systemic reforms may not reach their intended goals. Ensuring that teachers are both informed and supported in making instructional shifts is essential to advancing equitable and high-quality mathematics education worldwide.

Conclusions

Improving the quality of mathematics education and ensuring its alignment with the SDGs requires continuous adaptation to meet individual student needs and the broader societal demand for future jobs and a sustainable future. This study explored teachers' knowledge and perceptions of mathematical rigor—particularly the distinction between conceptual understanding and application—and their familiarity with the U.S. Standards for Mathematical Practice (SMPs). The findings revealed knowledge gaps, suggesting a need for targeted professional development to prepare teachers as effective agents of change.

Continuous improvement in education can be seen through teachers' ongoing professional development, as emphasized by the NCTM (2014), which recognizes teaching as a lifelong learning process. Schools must continuously support teachers to remain aligned with their evolving educational goals. This professional development ensures that teaching practices are responsive to the needs of the 21st-century workforce (Garvin, 2000). For continuous improvement to contribute to systemic change, individual efforts must be scaled up from classroom-level practices to influence school- and district-wide policy reforms (NCTM, 2014).

An essential aspect of continuous improvement is ensuring that all students benefit from these advancements, particularly those from underrepresented or marginalized groups. As part of the broader SDG4 objectives, continuous improvement in mathematics education must focus on closing achievement gaps and providing equitable access to high-quality instruction. This ensures that systemic changes address the needs of all learners alike (Leikin, 2011; Powell, 2015), particularly those disadvantaged by traditional instructional practices, such as rote memorization. Schools must leverage all educators' collective learning potential. By nurturing a culture of collaboration and continuous improvement, schools can empower teachers to drive change in mathematics education.

SDG4 underscores the importance of inclusive and equitable quality education, which is directly linked to improving teachers' knowledge of mathematics education standards and practices. Professional development initiatives focused on enhancing teachers' skills and content knowledge are critical for meeting SDG4 targets. Knowledge gaps among teachers serve as barriers to achieving quality education, and disparities in teacher preparation contribute to inequities in student experiences. Students taught by well-prepared teachers are more likely to receive high-quality instruction, highlighting the need to bridge these knowledge gaps and promote equitable educational outcomes for all students.

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